

Prospects for measuring the moment of inertia of pulsar J0737-3039A

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Abstract

Here we consider the possibility—envisaged by many authors as feasible in the near future—of measuring at 10% or better the moment of inertia I of the pulsar J0737-3039A via the gravitomagnetic spin-orbit periastron precession (analogous to the Lense-Thirring pericentre precession in the case of a test-particle orbiting a central spinning mass). Such a gravitomagnetic effect is expected to be of the order of 10^{-4} deg yr $^{-1}$ and the present-day precision in measuring the periastron precession of J0737-3039A via pulsar timing is 6.8×10^{-4} deg yr $^{-1}$. However the systematic uncertainty in the much larger first-order post-Newtonian (1PN) gravitoelectric precession (analogous to the Einstein Mercury’s perihelion precession in the weak-field and slow-motion approximation), which should be subtracted from the measured one in order to pick up the gravitomagnetic rate, is of primary importance. Indeed, by determining the sum of the masses by means of the third Kepler law, such a bias amounts to 0.03165 deg yr $^{-1}$, according to the current level of accuracy in knowing the parameters of the J0737-3039 system. The major sources of uncertainty are the Keplerian projected semimajor axis x_B of the component B and the post-Keplerian parameter s , identified with $\sin i$; their knowledge should be improved by three orders of magnitude at least; the bias due to the Keplerian projected semimajor axis x_A of the component A amounts to $\approx 10\%$ today. The present-day level of accuracy in the eccentricity e would affect the investigated measurement at a percent level, while the impact of the orbital period P_b is completely negligible. If, instead, the sum of the masses is measured by means of the post-Keplerian parameters r and s , it turns out that r should be measured five orders of magnitude better than now: according to the present-day level of accuracy, the total uncertainty in the 1PN periastron rate is, in this case, 2.11819 deg yr $^{-1}$. In conclusion, the prospect of measuring the moment of inertia of PSR J0737-3039A at 10% accuracy or better seems unlikely given the limitations to the precision with which the system’s basic binary and post-Keplerian parameters can be measured via radio timing.

Key words: binaries; pulsars: general—pulsars: individual, (PSR J0737-3039A/B): moment of inertia

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1. Introduction

The measurement of the moment of inertia I of a neutron star at a 10% level of accuracy or better would be of crucial importance for effectively constraining the Equation-Of-State (EOS) describing matter inside neutron stars (Morison et al., 2004; Bejger et al., 2005; Lattimer and Schutz, 2005; Lavagetto et al., 2007).

After the discovery of the double pulsar PSR J0737-3039A/B system (Burgay et al., 2003), whose relevant orbital parameters are listed in Table 1, it was often argued that such a measurement for the A pulsar via the post-Newtonian gravitomagnetic spin-orbit periastron precession (Barker and O’Connell, 1975a; Damour and Schaefer, 1988; Wex, 1995) would be possible after some years of

accurate and continuous timing. Lyne et al. (2004) write: “Deviations from the value predicted by general relativity may be caused by contributions from spin-orbit coupling (Barker and O’Connell, 1975b), which is about an order of magnitude larger than for PSR B1913+16. This potentially will allow us to measure the moment of inertia of a neutron star for the first time (Damour and Schaefer, 1988; Wex, 1995).”

According to Lattimer and Schutz (2005), “measurement of the spin-orbit perihelion advance seems possible.”

In (Kramer et al., 2006) we find: “A future determination of the system geometry and the measurement of two other PK parameters at a level of precision similar to that for $\dot{\omega}$, would allow us to measure the moment of inertia of a neutron star for the first time (Damour and Schaefer, 1988; Wex, 1995). [...] this measurement is potentially very difficult [...] The double pulsar [...] would also give insight into the nature of super-dense matter.”

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In (Damour, 2007) it is written: “It was then pointed out (Damour and Schaefer, 1988) that this gives, in principle, and indirect way of measuring the moment of inertia of neutron stars [...]. However, this can be done only if one measures, besides ¹ k , two other PK parameters with 10^{-5} accuracy. A rather tall order which will be a challenge to meet.”

Some more details are released by Kramer et al. (2005): “[...] a potential measurement of this effect allows the moment of inertia of a neutron star to be determined for the first time (Damour and Schaefer, 1988). If two parameters, e.g. the Shapiro parameter s and the mass ratio R , can be measured sufficiently accurate, an expected $\dot{\omega}_{\text{exp}}$ can be computed from the intersection point.”

Here we wish to examine, more precisely and with more quantitative details than done in the existing literature, the conditions which would make feasible to measure I_A at 10% or better in the PSR J0737-3039A/B system in view of the latest timing results (Kramer et al., 2006). In particular, we will show how crucial the impact of the mismodelling in the known precessional effects affecting the periastron rate of PSR J0737-3039A/B is if other effects are to be extracted from such a post-Keplerian parameter. Such an analysis will turn out to be useful also for purposes other than measuring gravitomagnetism like, e.g., putting more severe and realistic constraints on the parameters entering various models of modified gravity. Indeed, in doing that for, e.g., a uniform cosmological constant Λ Jetzer and Sereno (2006) took into account only the least-square covariance sigma of the estimated periastron rate ($6.8 \times 10^{-4} \text{ deg yr}^{-1}$): the systematic bias due to the first post-Newtonian (1PN) periastron precession was neglected.

2. The systematic uncertainty in the 1PN periastron precession

By assuming $I \approx 10^{38} \text{ kg m}^2$ (Morrison et al., 2004; Berger et al., 2005), the gravitomagnetic spin-orbit periastron precession is about $\dot{\omega}_{\text{GM}} \approx 10^{-4} \text{ deg yr}^{-1}$, while the error $\delta\dot{\omega}_{\text{meas}}$ with which the periastron rate is phenomenologically estimated from timing data is currently $6.8 \times 10^{-4} \text{ deg yr}^{-1}$ (Kramer et al., 2006). In order to measure the gravitomagnetic effect—and, in principle, any other dynamical feature affecting the periastron— $\delta\dot{\omega}_{\text{meas}}$ is certainly of primary importance, but it is not the only source of error to be carefully considered: indeed, there are other terms contributing to the periastron precession (first and second post-Newtonian, quadrupole, spin-spin (Barker and O’Connell, 1975a; Damour and Schaefer, 1988; Wex, 1995)) which must be subtracted from $\dot{\omega}_{\text{meas}}$, thus introducing a further systematic uncertainty due to the propagation of the errors in the system’s parameters entering their analytical expressions. A preliminary analysis of such aspects, can be found in (Lattimer and Schutz, 2005). However, apart

from the fact that its authors make use of the value for i measured with the scintillation method (Coles et al., 2005) which is highly uncertain for the reasons given below, in using the third Kepler law to determine the sum of the masses they also confound the relative projected semimajor axis ² $a \sin i$ (see eq. (3)) with the barycentric projected semimajor axis x , which is the true measurable quantity from timing data, so that their analysis cannot be considered reliable. The semimajor axis a of the relative motion of A with respect to B in a binary system is just the sum of the semimajor axes a_{bc} of A and B with respect to the system’s barycenter, i.e. $a = a_{\text{bc}}^A + a_{\text{bc}}^B$.

Let us, now, consider the largest contribution to the periastron rate, i.e. the 1PN precession (Damour and Deruelle, 1986; Damour and Taylor, 1992)

$$\dot{\omega}_{\text{1PN}} = \frac{3}{(1 - e^2)} \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3}, \quad (1)$$

where $T_\odot = GM_\odot/c^3$ and $M = m_A + m_B$, in Solar masses,. It is often referred to as gravitoelectric in the weak-field and slow-motion approximation: in the context of the Solar System it is the well known Einstein Mercury’s perihelion precession of about 43 arcsec cy^{-1} . Thus,

$$\begin{cases} \dot{\omega}_{\text{GM}} = \dot{\omega}_{\text{meas}} - \dot{\omega}_{\text{1PN}} - \dot{\omega}_{\text{2PN}}, \\ \delta\dot{\omega}_{\text{GM}} \leq \delta\dot{\omega}_{\text{meas}} + \delta\dot{\omega}_{\text{1PN}} + \delta\dot{\omega}_{\text{2PN}} \end{cases} \quad (2)$$

The sum of the masses M enters eq. (1); as we will see, this implies that the relative semimajor axis a is required as well. For consistency reasons, the values of such parameters used to calculate eq. (1) should have been obtained independently of the periastron rate itself. We will show that, in the case of PSR J0737-3039A/B, it is possible.

Let us start from the relative semimajor axis

$$a = (1 + R) \left(\frac{cx_A}{\sin i} \right) = 8.78949386 \times 10^8 \text{ m}. \quad (3)$$

It is built in terms of R , the projected semimajor axis x_A and $\sin i$; the phenomenologically estimated post-Keplerian parameter s determining the shape of the logarithmic Shapiro time delay can be identified with $\sin i$ in general relativity and the ratio $R = x_B/x_A$ has been obtained from the phenomenologically determined projected semimajor axes, being equal to the ratio of the masses in any Lorentz-invariant theory of gravity (Damour and Deruelle, 1985; Damour and Schaefer, 1988; Damour and Taylor, 1992)

$$R = \frac{m_A}{m_B} + \mathcal{O} \left(\frac{v^4}{c^4} \right). \quad (4)$$

The uncertainty in a can be conservatively evaluated as

$$\delta a \leq \delta a|_R + \delta a|_s + \delta a|_{x_A}, \quad (5)$$

¹ The parameter k is directly related to the periastron precession $\dot{\omega}$.

² Our a must not be confused with the one used in (Lattimer and Schutz, 2005) which is, in fact, a_{bc} .

with

$$\begin{cases} \delta a|_R \leq \left(\frac{c x_A}{s}\right) \delta R = 466,758 \text{ m}, \\ \delta a|_s \leq a \left(\frac{\delta s}{s}\right) = 342,879 \text{ m}, \\ \delta a|_{x_A} \leq a \left(\frac{\delta x_A}{x_A}\right) = 621 \text{ m}. \end{cases} \quad (6)$$

Thus,

$$\delta a \leq 810,259 \text{ m}. \quad (7)$$

eq. (7) yields a relative uncertainty of

$$\frac{\delta a}{a} = 9 \times 10^{-4}. \quad (8)$$

It is important to note that x_B , via R , and s have a major impact on the overall uncertainty in a ; our estimate has to be considered as conservative because we adopted for δs the largest value quoted in (Kramer et al., 2006). In regard to the inclination, we did not use the more precise value for i obtained from scintillation measurements in³ (Coles et al., 2005) because it is inconsistent with that derived from timing measurements (Kramer et al., 2006). Moreover, the scintillation method is model-dependent and it is not only based on a number of assumptions about the interstellar medium, but it is also much more easily affected by various other effects. However, we will see that also x_A has a non-negligible impact. Finally, let us note that we purposely linearly summed up the individual sources of errors in view of the existing correlations among the various estimated parameters (Kramer et al., 2006).

Let us, now, determine the sum of the masses: recall that it must not come from the periastron rate itself. One possibility is to use the phenomenologically determined orbital period P_b and the third Kepler law getting⁴

$$GM = a^3 \left(\frac{2\pi}{P_b}\right)^2. \quad (9)$$

With eq. (3) and eq. (9) we can, now, consistently calculate eq. (1) getting

$$\dot{\omega}_{1\text{PN}} = \frac{3}{(1-e^2)} \left(\frac{x_A + x_B}{s}\right)^2 \left(\frac{2\pi}{P_b}\right)^3 = 16.90410 \text{ deg yr}^{-1}; \quad (10)$$

in this way the 1PN periastron precession is written in terms of the four Keplerian parameters P_b , e , x_A , x_B and of the post-Keplerian parameter s . The mismodeling in them yields

$$\begin{cases} \delta \dot{\omega}_{1\text{PN}}|_{x_B} \leq 2 \dot{\omega}_{1\text{PN}} \left[\frac{\delta x_B}{(x_A + x_B)}\right] = 0.01845 \text{ deg yr}^{-1}, \\ \delta \dot{\omega}_{1\text{PN}}|_s \leq 2 \dot{\omega}_{1\text{PN}} \left(\frac{\delta s}{s}\right) = 0.01318 \text{ deg yr}^{-1}, \\ \delta \dot{\omega}_{1\text{PN}}|_{x_A} \leq 2 \dot{\omega}_{1\text{PN}} \left[\frac{\delta x_A}{(x_A + x_B)}\right] = 1 \times 10^{-5} \text{ deg yr}^{-1}, \\ \delta \dot{\omega}_{1\text{PN}}|_e \leq 2e \dot{\omega}_{1\text{PN}} \left[\frac{\delta e}{(1-e^2)}\right] = 2 \times 10^{-6} \text{ deg yr}^{-1}, \\ \delta \dot{\omega}_{1\text{PN}}|_{P_b} \leq 3 \dot{\omega}_{1\text{PN}} \left(\frac{\delta P_b}{P_b}\right) = \mathcal{O}(10^{-8}) \text{ deg yr}^{-1}. \end{cases} \quad (11)$$

Thus, the total uncertainty is

$$\delta \dot{\omega}_{1\text{PN}} \leq 0.03165 \text{ deg yr}^{-1}, \quad (12)$$

which maps into a relative uncertainty of

$$\frac{\delta \dot{\omega}_{1\text{PN}}}{\dot{\omega}_{1\text{PN}}} = 1.8 \times 10^{-3}. \quad (13)$$

As a consequence, we have the important result

$$\Delta \dot{\omega} \equiv \dot{\omega}_{\text{meas}} - \dot{\omega}_{1\text{PN}} = (-0.00463 \pm 0.03233) \text{ deg yr}^{-1}. \quad (14)$$

Every attempt to measure or constrain effects predicted by known Newtonian and post-Newtonian physics (like, e.g., the action of the quadrupole mass moment or the gravitomagnetic field), or by modified models of gravity, for the periastron of the PSR J0737-3039A/B system must face with the bound of eq. (14).

Should we decide to use both the post-Keplerian parameters related to the Shapiro delay (Damour and Deruelle, 1986; Damour and Taylor, 1992)

$$\begin{cases} r = T_{\odot} m_B, \\ s = x_A \left(\frac{P_b}{2\pi}\right)^{-2/3} T_{\odot}^{-1/3} M^{2/3} m_B^{-1}, \end{cases} \quad (15)$$

for determining the sum of the masses, we would have, with

$$\dot{\omega}_{1\text{PN}} = \frac{3}{(1-e^2)} \left(\frac{P_b}{2\pi}\right)^{3/2} \left(\frac{r}{x_A}\right)^{9/4} \frac{s^{19/4}}{(x_A + x_B)^{5/2}}, \quad (16)$$

which yields

$$\dot{\omega}_{1\text{PN}} = 17.25122 \pm 2.11819 \text{ deg yr}^{-1}. \quad (17)$$

The major source of uncertainty is r , with $2.06264 \text{ deg yr}^{-1}$; the bias due to the other parameters is about the same as in the previous case.

³ See also (Lyutikov and Thompson, 2005).

⁴ In principle, also the 1PN correction to the third Kepler law calculated by (Damour and Deruelle, 1986) should be included, but it does not change the error estimate presented here.

Let us, now, consider the second post-Newtonian contribution to the periastron precession (Damour and Schaefer, 1988; Wex, 1995)

$$\dot{\omega}_{\text{2PN}} = \frac{3(GM)^{5/2}}{c^4 a^{7/2} (1-e^2)^2} \left\{ \left[\frac{13}{2} \left(\frac{m_A^2 + m_B^2}{M^2} \right) + \frac{32}{3} \left(\frac{m_A m_B}{M^2} \right) \right] \right\}, \quad (18)$$

up to terms of order $\mathcal{O}(e^2)$. For our system it amounts to $4 \times 10^{-4} \text{ deg yr}^{-1}$, so that it should be taken into account in $\Delta\dot{\omega}$. However, it can be shown that the bias induced by the errors in M and a amounts to $4 \times 10^{-6} \text{ deg yr}^{-1}$, thus affecting the gravitomagnetic precession at the percent level.

3. Discussion and conclusions

O’Connell (2004), aware of the presence of other non-gravitomagnetic contributions to the pulsar’s periastron rate, proposed to try to measure the gravitomagnetic spin-orbit precession of the orbital angular momentum (Barker and O’Connell, 1975a) (analogous to the Lense-Thirring node precession in the limit of a test particle orbiting a massive body) which is not affected by larger gravitoelectric contributions. However, its magnitude is $\approx (10^{-4} \text{ deg yr}^{-1}) \sin \psi$, where ψ is the angle between the orbital angular momentum and the pulsar’s spin; thus, it would be negligible in the PSR J0737-3039A/B system because of the near alignment between such vectors (Stairs et al., 2006), in agreement with the observed lack of profile variations (Manchester et al., 2005; Kramer et al., 2006).

In regard to the measurement of the moment of inertia of the component A via the gravitomagnetic periastron precession, our analysis has pointed out that the bias due to the mismodelling in the 1PN gravitoelectric contribution to periastron precession—expressed in terms of the phenomenologically measured parameters P_b, e, x_A, x_B, s —is the most important systematic error exceeding the expected gravitomagnetic rate, at present, by two orders of magnitude; the major sources of uncertainty in it are x_B and s , which should be measured three orders of magnitude better than now to reach the 10% goal. The projected semimajor axis x_A of A, if known one order of magnitude better than now, would induce a percent-level bias. Instead, expressing the 1PN gravitoelectric periastron rate in terms of P_b, e, x_A, x_B, s, r would be definitely not competitive because the improvement required for r would amount to five orders of magnitude at least. We prefer not to speculate now about the size of the improvements in timing of the PSR J0737-3039A/B system which could be achieved in future. Since the timing data of B are required as well for x_B and in view of the fact that B appears as a strong radio source only for two intervals, each of about 10-min duration, while its pulsed emission is rather weak or even undetectable for most of the remainder of the orbit (Lyne et al., 2004; Burgay et al., 2005), the possibility of reaching in a near future the required accuracy to effectively constrain I_A to 10% level or better should be considered with more

skepticism than done so far. Measuring gravitomagnetism is a challenging enterprise.

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Table 1

Relevant Keplerian and post-Keplerian parameters of the binary system PSR J0737-3039A/B (Kramer et al., 2006). The orbital period P_b is measured with a precision of 4×10^{-6} s. The projected semimajor axis is defined as $x = (a_{bc}/c) \sin i$, where a_{bc} is the barycentric semimajor axis (the relative semimajor axis $a = (x_A + x_B)c / \sin i$), c is the speed of light and i is the angle between the plane of the sky, perpendicular to the line-of-sight, and the orbital plane. The eccentricity is e . The best determined post-Keplerian parameter is, to date, the periastron rate $\dot{\omega}$ of the component A. The phenomenologically determined post-Keplerian parameter s , related to the general relativistic Shapiro time delay, is equal to $\sin i$; we have conservatively quoted the largest error in s reported in (Kramer et al., 2006). The other post-Keplerian parameter related to the Shapiro delay, which is used in the text, is r .

P_b (d)	x_A (s)	x_B (s)	e	$\dot{\omega}$ (deg yr $^{-1}$)	s	r (μ s)
0.10225156248(5)	1.415032(1)	1.5161(16)	0.0877775(9)	16.89947(68)	0.99974(39)	6.21(33)